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Applications of High-Frequency Gravitational Waves (HFGWs)

Robert M. L. Baker, Jr.

GRAVWAVE® LLC and Transportation Sciences Corporation, 8123 Tuscany Avenue Playa del Rey, California 90293-7856*, USA,*
(310) 823-4143, Fax (310) 821-1694, robert.baker.jr@comcast.net

Abstract. Applications to space technology of High-Frequency Gravitational Waves, (HFGWs), defined as having frequencies in excess of 100 kHz, are discussed. The applications to be specifically addressed include: providing (1) multi-channel communications (both point to point and point to multipoint through all normal material things – the ultimate wireless system); (2) a remote means for causing perturbations to the motion of objects such as missiles (bullets to ICBMs), spacecraft, land or water vehicles or craft; (3) remote coalescing of clouds of hazardous vapors, radioactive dust, etc. by changing the gravitational field in their vicinity; (4) the potential for through-earth or through-water "X-rays" in order to observe subterranean structures, geological formations, create a transparent ocean, view three-dimensional building interiors, buried devices, etc.; and (5) the potential for remotely disrupting the gravitational field in a specific region of space. The utilization of a possible HFGW telescope as a navigational aid by viewing the anisotropic or patterned HFGW relic cosmic background above, on, or under the ground without reliance on GPS satellite signals is also noted. Many of the applications are discussed in the context of space technology and several approaches to the generation and possible focusing of HFGWs are referenced. A derivation of the "jerk" formulation of the quadrupole approximation for HFGW power is included in an appendix.

INTRODUCTION

The general objective of the devices whose applications are discussed in this paper is to emulate scientifically acceptable generation of gravitational waves (GWs) like those that are produced by energizable celestial systems such as rotating binary stars, star-black-hole collisions, binary black holes, asymmetrical spinning or pulsating black holes, etc. through the use of smaller macro- and micro-, terrestrial or laboratory energizable systems. Such terrestrial systems generate well over 35 orders of magnitude more force intensity by virtue of their use of non-gravitational forces (nuclear or electromagnetic compared to gravitational; please see Baker, (2000) than a typical celestial system and well over 12 orders of magnitude greater frequency compared to most celestial sources (having frequencies of a kHz to very small fractions of a Hz). High-Frequency Gravitational Waves, (HFGWs), are defined as having frequencies in excess of 100 kHz, according to Hawking and Israel (1979). Terrestrial energizable systems produce significant and useful GW according to the various designs of the devices to be described at this Forum, even though they are orders of magnitude smaller and less massive than the extraterrestrial celestial systems. Such devices could involve myriads of piezoelectric resonators, tabletop X-ray lasers, ultra-small electromagnets, the Gertsenshtein effect, etc. Examples of proposed HFGW laboratory generators include Baker 2000; 2003a; 2004b; 2005; Baker and Li, 2005; Fontana and Baker, 2003; Li, 2003; Rudenko, 2003; Stephenson, 2005; and Woods and Baker, 2005.

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These applications to space technology are discussed under the broad categories of communications, propulsion, astronomy, imagery, and physics.

THE QUADRUPOLE APPROXIMATION

In an appendix is to be found the derivation of the "jerk" formulation of the quadrupole equation originally derived by Einstein (1916) is found. Such a formulation is central to many of the devices that have been proposed to generate HFGW in the laboratory and lead to the applications described herein. The quadrupole equation is an approximation to the *power* of GWs in *weak fields* that are generated by a rapid change in acceleration of a mass (conventionally referred to as a "jerk" or alternatively referred to as a "shake" or a "jolt"). The quadrupole is the lowest-order solution to the GW propagation problem and mass motions that have quadrupole moments are the most effective GW generators. That is, the quadrupole itself is not the physical process at all, but only one means of establishing the power of the gravitational wave – the lowest-order solution (Einstein, 1916; Einstein and Rosen, 1937). There are, of course, other means to generate GWs besides mass motion, for example the Gertsenshtein (1962) EM to GW effect. As derived in the Appendix the quadrupole approximation to GW power can be phrased as:

$$P(r, \Delta f, \Delta t) = 1.76\text{x}10^{-52}\,(2r\Delta f/\Delta t)^2 \quad \text{W}, \tag{1}$$

which is the *jerk formulation of the quadrupole equation* as developed in Baker (2000) and in U. S. Patent 6,417,587. In the equation r is the radius of gyration of a system of masses, m, Δf is the change in force, N, over the incremental time interval Δt, s. For a constant mass, δm, $\Delta f/\Delta t = \delta m\,\Delta(\text{acceleration})/\Delta t$, so that the equation states that a third time derivative is imparted to the motion of the mass (commonly defined as a "jerk"), or energizable element, such as a piezoelectric membrane resonator, laser target, permanent magnet, particles such as electrons, etc.**.** For a continuous train of jerks the frequency, ν, is $\nu = 1/\Delta t$, and Eq. (1) can be phrased as a function of HFGW frequency as

$$P(r, \Delta f, \nu) = 1.76\text{x}10^{-52}\,(2r\nu\Delta f)^2 \quad \text{W}. \tag{2}$$

The energizable elements are energized by energizing elements such as electrical and/or magnetic fields, X-ray lasers, electromagnets, etc. in weak fields.

Three important points should now be made:

(1) Energizing/Energizable element's Action/Reaction does not cancel out during GW generation: Einstein and Rosen (1937); but the energizable elements should be asymmetrically distributed.

(2) The weak field can be well over 100 g's: e.g., as computed in Baker (2000) and U. S. Patent 6,417,597. The weak-field acceleration of PSR 1913 + 16 is 112 g's at periastron. Analyses of a double-pulsar star system (Lyne, et al. 2004) may show that much larger g forces would not greatly reduce the quadrupole's accuracy. Also the size of the energizable elements should be smaller than the GW wavelength. On the other hand, Grischuk (2003) suggested that the requirement for the dimensions of the energizable element and/or r « λ_{GW} may ***not*** *be a stringent or even a necessary one. Both requirements are for the quadrupole approximation to the GW power to hold accurately.*

(3) Δf need NOT be gravitational force (please see Einstein,1916; Infeld quoted by Weber 1964, p. 97). Electromagnetic forces are more than 10^{35} larger than gravitational forces and should be employed in laboratory GW generation. As Weber (1964, p.97) points out: "The nongravitational forces play a decisive role in methods for detection and generation of gravitational waves ..."

APPLICATIONS

For the engineer the prospect for applying HFGW technology is more exciting and compelling than even the prospect for the epoch-making experiment to generate and detect HFGW in the laboratory. In the following subsections the applications of HFGW will quite generally be addressed regarding communications, propulsion, imaging, astronomy, and physics. These sections are designed to show how applicable this technology is to the military, aerospace or civilian sectors. The HFGW generator experiment is, of course, crucial to the success of these applications.

Communication

Like the gravitational field itself, GW passes unattenuated through all normal matter and can, for example, reach deeply submerged submarines or pass directly through the Earth. This feature also represents a new paradigm for interplanetary communications. As Thomas Prince (Chief Scientist, *NASA/JPL* and Professor of Physics at *Caltech*) recently commented (2002): "Of the applications (of HFGWs), communication would seem to be the most important. Gravitational waves have a very low cross section for absorption by normal matter and therefore high-frequency waves could, in principle, carry significant information content with effectively no absorption unlike any electromagnetic waves." Interestingly, some analyses have shown that a HFGW transmitter could become a receiver and vice versa; however, the theoretical considerations need to be validated experimentally. Such a HFGW communication system would represent the ultimate wireless system -- point-to-multipoint PHz communication without the need for expensive enabling infrastructure. Thus there would be no need for fiber-optic cable, satellite transponders, microwave relays, etc. Antennas, cables, and phone lines would be a thing of the past!

Propulsion

Landau and Lifshitz (1975), on page 349 of their internationally recognized authoritative treatise state: "Since it has definite energy, the gravitational wave is itself the source of some additional gravitational field... its field is a second-order effect ... But *in the case of high-frequency gravitational waves the effect is significantly strengthened ...*" Thus it is possible to change the gravitational field near an object or objects by means of HFGWs and move them. Dott. Giorgio Fontana (2003), quotes theories that predict HFGWs can be employed for propulsion, that is, the generation of space-time singularities with colliding beams of HFGWs (please also see Ferrari (1988)) and could be a form of "propellantless propulsion." The concept (Baker, 2001)is that HFGW energy beamed from off board can create gravitational distortions, that is, "Hills" and "Valleys" in the spacetime continuum that the spacecraft or other vehicle is repelled by, or "falls into," or "falls toward." Again, HFGW is not only proposed as propulsion means, but also as a means to modify the trajectories of moving objects such as bullets, missiles, or spacecraft! One could also cause other objects above, on, or below the ground or water to move. The ability to change a gravitational field remotely could also lead to the coalescing of clouds of hazardous vapors, radio-active particles, etc.

Astronomy

As discussed by Baker (2003b; 2004a) and Baker, Davis, and Woods (2005) lower diffraction for HFGW allows for imaging using the possible refractive properties of a High-Temperature Superconductor or HTSC for use in communications, propulsion, and optics. Such refractive properties were first discussed by Ning Li and David G. Torr (1992). This paper was peer reviewed and examined by C. A. Lundqist and Jeeva Anandan, but their results are disputed by Kowitt (1994). Theoretically, one can image (resolve) two point sources whose angular separation, α_d, (the diffraction limit) is given by

$$\alpha_d \quad = \quad 1.22\lambda_{GW}/D \quad \text{radians,} \tag{3}$$

where D is the diameter of the aperture of the optical device. Thus the smaller the gravitational wavelength, $\lambda_{GW,}$ is (and the higher the HFGW frequency is) the greater the resolution. The possible refractive (Baker, 2003b) and Fresnel-effect reflective (Baker, Davis, and Woods, 2005) properties of HTSC could also open up the possibility of a HFGW Refracting and/or Reflecting Telescope. But the optical properties of a HFGW pulse train and refractive properties of HTSCs are speculative and require further study. By utilizing such telescopes it may, however, be possible to intensify the very slightly anisotropic relic HFGW cosmic background features (arising before the cosmos was translucent) that may exist (MHz to THz) and possibly image HFGW celestial point sources such as rapid stellar compression shock waves (jerks) and even more speculative, evaporating, relic mini black holes that are relatively nearby – a candidate for *Dark Matter* (as mentioned to me by John Miller on May 4, 2002). Therefore, the *Big Crunch/Big Bang* theories developed by Nickolas Gorkavyi (2003) could be tested. Because the relic cosmic background is patterned (Brustein, 1995; Hogan, 2002) it could be utilized for navigational fixes like the stars and lead to a GPS-free navigation system virtually *anywhere in or under the Earth*!

Imaging

If intervening matter between the HFGW generator and detector causes a change (even a very slight one) in HFGW polarization, direction (refraction), frequency (dispersion) or results in extremely slight scattering or absorption, then it may be possible to develop a HFGW "X-ray" like system. It may, in fact, be possible to image directly through the Earth or water, create a transparent ocean, and view subterranean features in three dimensions, such as geological ones, submerged or buried devices, or underground or above ground real-time 3D building interior views, to a sub-millimeter resolution (for THz HFGW) as was discussed in Baker (2003b; 2004a).

Physics

When high-intensity HFGW is focused by means of a High-Temperature Superconductor or HTSC lenses or, possibly, mirrors to dimensions of a few microns or less, GW-driven nuclear phenomena, including quantum jitter, may occur. As discussed in U. S. Patents 6,160,336 and 6,784,591 and section 6 of Baker (2003a) it is hypothesized (along with Greene, 1999) that since the smaller particles have a more detailed structure they are more fragile and susceptible to space-time geometry warp or tear caused by gravitational stress related to a large gravitational-energy or HFGW flux. It is also recognized (Ledingham, *et al.* 2003) that as the focused intensity of lasers (or possibly HFGW) increases from 10^{20} to 10^{26} Wm^{-2} photoexcitation of low-lying nuclear levels, photonuclear and ion-induced reactions, and pion production can occur. By utilizing speculative HTSC focal-spot enclosures, lenses and mirrors for concentrating HFGW power, the theoretical, non-gravitational-force generated HFGW *ten-MW* pulse output predicted in Fontana and Baker (2003), *380-kW* continuous power predicted in Baker (2003a), and the *11-kW* continuous power predicted in Li (2003), could provide HFGW fluxes in excess of 10^{19} Wm^{-2} (Baker and Li, Table 1, 2005) and remotely disrupt the gravitational field in a specific region of space and possibly cause nuclear reactions there.

CONCLUSIONS

The application of HFGW to aerospace communications, propulsion, astronomy, imaging, and physics has been briefly described. It should, however, be borne in mind that many other applications may be discovered after the successful completion of a HFGW generation and detection experiment. Certainly Marconi never envisioned the application of his extremely low power, spark-gap generated, ship/shore radio-telegraph apparatus to microwave ovens, radar, cell phones, or television. We may well be surprised after we are able to generate and detect HFGW in the laboratory.

NOMENCLATURE

a	=	semi-major axis of a two-body orbit (AU)
c	=	speed of light, $2.998\text{x}10^8$ (ms^{-1})
D	=	diameter of aperture (m)
$D_{\alpha\beta}$	=	quadrupole moment-of-inertia tensor
E	=	energy (J)
e	=	eccentricity of a two-body orbit
f	=	force (N)
$\mathbf{f_{cf}}$	=	centrifugal-force vector (N)
G	=	universal gravitational constant = $6.67423\text{x}10^{-11}$ ($m^3/kg\text{-}s^2$)
I	=	moment of inertia ($kg\text{-}m^2$)
M	=	mean anomaly for a two-body orbit (radians or degrees)
M	=	mass of an object (kg)
n	=	mean motion (radians s^{-1})
P	=	magnitude of the power of a gravitational-radiation source (W)
p	=	parameter or semilatus rectum = $a(1-e^2)$ (AU)
q	=	periastron distance = a(1-e) (AU)

r = radial distance to an object; alternately, the effective radius of gyration (m)
t = time (s)
$t`$ = spinning- rod time (s)
v = true anomaly of a two-body orbit (radians or degrees)
α = diffraction angle (radians or degrees)
Δ = small increment
$\Delta f_{cf\ x}$ = incremental x component of centrifugal force (N)
Δf_{cfy} = incremental y component of centrifugal force (N)
Δt = time increment (s)
δm = differential mass (kg)
δt = differential time interval (s)
θ = the central angle of a rotating rod (radians or degrees)
λ = wavelength (m)
μ = $m_1 + m_2$ = sum of masses on a two-body orbit in characteristic units
τ = characteristic time; for heliocentric unit systems = 5.022×10^5 (s)
ω = angular rotational rate (rad/s)

Subscripts

1 refers to mass one
2 refers to mass two
cf centrifugal
d diffraction
GW gravitational wave
t tangential
x x component
y y component

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APPENDIX

Jerk, or Third Time Derivative of the Motion of a Mass, Formulation of the Quadrupole Equation

There is no new Physics here, simply a different approach or formulation of the conventional equations utilized to estimate GW power in order to render engineering applications more apparent. The standard quadrupole equation, which was originally formulated by Einstein in 1916, to compute the HFGW power, will be employed. The basic quadrupole approximation will be defined in terms of a change in force, Δf, over a short time interval, Δt, which is defined as a "***jerk.***" The derivation of this basic jerk equation will be accomplished by two separate analysis paths: one starting with the third derivative of the moment of inertia formulation of the quadrupole equation and the other starting with the spinning rod (or binary-star orbit) formulation of the quadrupole equation. The resulting jerk equation will be numerically checked against the known result for pulsar PSR 1913 + 16.

Derivation from Third Time Derivative of the Moment of Inertia

As is well known and noted specifically in a letter from Dr. Geoff Burdge (2000), Deputy Director for Technology and Systems of the *National Security Agency*: "Because of symmetry, the quadrupole moment can be related to a principal moment of inertia, I, of a three-dimensional tensor of the system and … can be approximated by

$$-dE/dt \approx -G/5c^5 (d^3I/dt^3)^2 = -5.5\times10^{-54} (d^3I/dt^3)^2.\text{"} \quad \text{(A1a)}$$

or from Eq. (110.16), p. 355 of Landau and Lifshitz (1975) :

$$P = \left| -dE/dt \right| = \kappa (G/45c^5)(d^3D_{\alpha\beta}/dt^3)^2 \ \text{W} \quad \text{(A1b)}$$

or

$$P = 1.76\times10^{-52} (d^3I/dt^3)^2 \qquad \text{W.} \quad \text{(A1c)}$$

This is Einstein's ***quadrupole equation***. In Eq. (A1a), k in Burdge's notation is G and the units in Eq. (A1c) are in the MKS system. In order to introduce the jerk concept let us consider the hypothetical example of an asymmetrical rim that, like an unbalanced ratchet wheel of a mechanical watch, need not be uniformly rotating or, in fact, not rotating at all. In this case, for a collection of masses, which are small energizable elements, non-uniformly distributed along the rim (e.g., nano-scale metal electrode elements of a piezoelectric resonator, laser targets, submicroscopic permanent magnets, microscopic electromechanical systems or MEMS, etc.)

$$I = \delta m\, r^2 \quad \text{kg-m}^2, \quad \text{(A2)}$$

where

δm = mass of an individual energizable elements along the rim, kg, and

r = the distance from a pivot out to any single δm on the rim, m, (or more exactly, the *radius of gyration* of the asymmetrical or segmented rim). Thus

$$d^3I/dt^3 = \delta m d^3 (r^2)/dt^3 = 2r\delta m(d^3r/dt^3) + \ldots \quad \text{(A3a)}$$

Approximately, by delta differentiation,

$$2r\{\delta m(d^3r/dt^3)\} \approx 2r\{\delta m\Delta(d^2r/dt^2)/\Delta t\} \quad \text{(A3b)}$$

and, by noting that by Newton's second law of motion,

$$f_t = \delta m(d^2r/dt^2), \quad \text{(A4a)}$$

we have, again by delta differentiation,

$$\delta m\Delta(d^2r/dt^2) = \Delta f_t \quad \text{(A4b)}$$

where f_t = tangential (to rim or ring or circle) force on δm and Δf_t is the rapid increase in f_t over time Δt (conventionally defined as *the jerk*). The third derivative of I is, therefore, approximated by

$$d^3I/dt^3 \cong 2r\, \Delta f_t/\Delta t\ , \quad \text{(A5)}$$

in which Δf_t is the nearly instantaneous *increase* in the force on piezoelectric metal electrode, X-ray-laser target, permanent magnet (or other energizable element) sites, δm, caused by Magnetrons, X-ray lasers, the magnetic field of current-carrying coils (or other energizing elements) when they are turned on and off or pulsed by a computer-control logic system resulting in a jerk. Let us now visualize a stack of stationary segmented or asymmetrical rims or rings; each one composed of a circle of non-uniformly distributed small energizable elements that are surrounded by close-by energizing elements. In this regard, the energizing elements adjacent to the periphery of each rim are sequenced (at the local GW speed, say the speed of light) along the stack of rims from one rim to the next in order to generate or build up the train of coherent HFGW as they move through the stack of rims (energizable element sites). In order *not* to build up acceleration the jerks are reciprocating; but due to the square in the kernel of the quadrupole equation, the GW radiates in a dumbbell-shaped pattern (Baker, Davis, and Woods, 2005) in both directions along the axis of the circular rims (through their centers) no matter which direction the energizable asymmetrically distributed elements are jerked. In summary, by substituting Eq. (A5) into Eq. (A1c),

$$P = 1.76\times10^{-52} (2r\Delta f_t/\Delta t)^2 \quad \text{W,} \quad \text{(A6)}$$

which is the *jerk formulation of the quadrupole equation.*

Derivation from a Spinning Rod

An alternative derivation of Eq. (A6) is as follows: From Eq. (1), p. 90 of Joseph Weber (1964) one has for Einstein's 1916 formulation of the gravitational-wave (GW) radiated power of a rod spinning about an axis through its midpoint having a moment of inertia, I, kg-m^2, and an angular rate, ω, radians/s, (also please see, for example, pp. 979 and 980 of Misner, Thorne, and Wheeler (1973) in which I in the kernel of the quadrupole equation also takes on its classical-physics meaning of an ordinary moment of inertia):

$$P = 32GI^2\,\omega^6/5c^5 = G(I\omega^3)^2/5(c/2)^5 \;,\quad \mathrm{W} \tag{A7}$$

or , with $I = r^2m$ (r being the radius of gyration of the rod),

$$P = 1.76\mathrm{x}10^{-52}(I\omega^3)^2 = 1.76\mathrm{x}10^{-52}(r\{rm\omega^2\}\omega)^2 \quad \mathrm{W} \tag{A8}$$

where $\{rm\omega^2\}$ can be associated with the magnitude of the rod's centrifugal-force vector, $\mathbf{f_{cf}}$. Equation (A8) is the *same equation as that given for two bodies on a circular orbit* on p. 356 of Landau and Lifshitz (1975) ($I=\mu r^2$ in their notation) where ω = n, the orbital mean motion, radians/s. The $\mathbf{f_{cf}}$ vector reverses every half period at *twice the angular rate of the rod* (and a $\mathbf{f_{cf}}$ component's magnitude completes one complete period in half the rod's period as in Weber (1964), p. 90). Thus the GW frequency is $2(\omega/2\pi)$, where ω is in radians/s. The change in the centrifugal-force vector itself (essentially a "jerk" when divided by a time interval) is a differential vector at right angles to the $\mathbf{f_{cf}}$ vector and directed tangentially along the arc that the dumbbell or rod moves through. The differential change in, for example, the x-component of the change in centrifugal force, Δf_{cfx}, is $f_{cfx}\,\Delta\theta$ and the change in the y-component is Δf_{cfy} is $f_{cfy}\,\Delta\theta$, where θ is the central angle of the rotating rod in radians. By delta differentiation of $f_{cf}^{\;2} = f_{cfx}^{\;2} + f_{cfy}^{\;2}$,

$$f_{cf}\,\Delta f_{cf} = f_{cfx}\Delta f_{cfx} + f_{cfy}\Delta f_{cfy} \tag{A9a}$$

and when one associates the components $\Delta f_{cfx,y}$ with $f_{cfx,y}\,\Delta\theta$ and, after dividing by $\Delta t`$ (t` being spinning-rod time), and noting that $\Delta\theta/\Delta t` = \omega$,

$$f_{cf}\,\Delta f_{cf} / \Delta t` = (f_{cfx}^{\;2} + f_{cfy}^{\;2})\omega \;. \tag{A9b}$$

Thus $\Delta f_{cf} / \Delta t` = f_{cf}\omega$ ; but $\Delta t` = ½\Delta t$ since the period of the GW is half the period of the rod, so that

$$2\,\Delta f_{cf} / \Delta t = f_{cf}\omega, \tag{A9c}$$

but $f_{cf} = \{rm\omega^2\}$ so

$$2\,\Delta f_{cf}/\Delta t = \{rm\omega^2\}\omega \tag{A9d}$$

and substituting Eq. (A9d) into Eq. (A8) yields

$$P = 1.76\mathrm{x}10^{-52}\,(2r\Delta f_{cf}\,/\Delta t)^2, \tag{A10}$$

where $(2r\Delta f_{cf}/\Delta t)^2$ is the kernel of the quadrupole approximation equation and $\Delta f_{cf}/\Delta t$ is, again, the *jerk.* Equation (A10) is *identical* to Eq. (A6), but arrived at by a different analysis path. Equation (A6), like Eqs. (A1), (A7), (A8) and (A10), are approximations for GW power and may only hold accurately for $r \ll \lambda_{GW}$ and for speeds of the GW generator components far less than the speed of light, c. Please see, for example, Pais (1982), p. 280 and Thorne (1987), p. 357. On the other hand, Leonid P. Grishchuk (2003) suggested that the requirement that $r \ll \lambda_{GW}$ may ***not*** be a stringent or even a necessary one for the quadrupole approximation to GW power to hold. And for plane transverse GWs to exist, $r \gg \lambda_{GW}$. Infact, Einstein and Rosen (1937), p. 54, state "At distances (r) ... great compared with the wavelengths (λ_{GW}'s), a progressive wave can be represented with good approximation ..." and on p. 349 of Landau and Lifshitz (1975) they indicate the utility of "... dimensions (of generator) large compared to λ_{GW} ..."

Validation Based on Orbit of PSR 1913+16

As a numerical validation of Eq. (A10), that is a validation of the use of a jerk to estimate gravitational-wave power, let us utilize the approach for computing the gravitational-radiation power of the neutron-star pair PSR 1913+16 observed by Taylor (1994) and Hulse to demonstrate the existence of GW. In the case of a binary star pair such as PSR 1913+16, the magnitude of the GW power, P, is computed from the quadrupole equation, which for two masses on orbit about one another is given, for example, by an equation on p.

356 of L. D. Landau and E. M. Lifshitz (1975) or Peters and Mathews (1963), the time-constant factor in the equation for P is

$$8G^4 m_1^2 m_2^2 \mu/(15c^5). \tag{A11}$$

The *time-variable factor* in P is a function of the true anomaly, v, and orbital eccentricity, e, as given in Landau and Lifshitz (1975) is

$$(1+e\cos v)^4([1+\{e/12\}\cos v]^2+e^2\sin^2 v)/(a[1-e^2])^5. \tag{A12}$$

In conventional astrodynamic/celestial-mechanics notation (please see Herrick (1971)) this factor (i.e., Eq. (A12)) is

$$p/r^6+(dr/d\tau)^2/12\mu r^4\,, \tag{A13}$$

where p is the orbital "parameter" or semilatus rectum (= $a\{1 - e^2\}$) in AUs, r is the radial distance between the two masses in AUs, τ is the characteristic time measured in $k_s$days or in units of $5.022\text{x}10^6$ s for a heliocentric-unit system (utilized by Taylor (1994) and others for PSR 1913+16), μ is the sum of the two masses, i. e., $\mu = m_1 + m_2$ solar masses, and as usual $G = 6.67423\text{x}10^{-11}$ $m^3/kg\text{-}s^2$, and c is the speed of light = $2.998\text{x}10^8$ m/s. Note that one AU (astronomical unit) = $1.496\text{x}10^{11}$ m. The GW power radiated, P, which causes a perturbation in the semi-major axis, a, (and an attendant secular decrease in the orbital period) is obtained by integrating the time-variable factor, Eq. (A13), over the orbital period using the mean anomaly, M, as independent variable, which is directly proportional to the time (that is, M = n {t-T}, where n is the mean motion (ω in Landau and Lifshitz's (1975) notation, p. 357), n= 2π/Period = $2\pi/2.79\text{x}10^4$ = $2.25\text{x}10^{-4}$ rad/s, and T is the time of periastron passage). The x and y average delta centrifugal force (or centrifugal-force jerks) component(s), $\Delta f_{cfx,y}$ (which will later be utilized to validate the fundamental jerk equation numerically) are both

$$man^2= (5.56\text{x}10^{30})(2.05\text{x}10^9)(2.25\text{x}10^{-4})^2 = 5.77\text{x}10^{32}\ \text{N} \tag{A14}$$

and divided by *m* yields the average centrifugal acceleration = 103.78 m/s^2 = 10.6 g's. At periastron, r = q = a(1-e) = $(2.05\text{x}10^9)(1-0.641) = 7.36\text{x}10^8$ m (with e = 0.641), the centrifugal acceleration is $q(dv/d\tau)^2$ where $dv/d\tau = \sqrt{(\mu p)}/r^2$ (please see Baker (1967), p. 13). In this latter case μ = 2.8 solar masses, a = 2.95 solar radii = $(2.95)(6.965\text{x}10^8$ m/solar radii)/$1.496\text{x}10^{11}$ m/AU = 0.01373 AU, p = $a\{1-e^2\}$ = 0.01373{1- 0.4109} = 0.00809 AU, and q = r = $7.36\text{x}10^8$ m/$1.496\text{x}10^{11}$m/AU = 0.00495 AU. After inserting these numbers we have $dv/d\tau = (\sqrt{[2.8\text{x}0.00809]/[0.00495]^2})/5.022\text{x}10^6$ s/$k_s$day = $1.223\text{x}10^{-3}$ radians/s. Thus the centrifugal *acceleration* at periastron of the star pair is $q(dv/dt)^2 = (7.36\text{x}10^8\ \text{m})(1.223\text{x}10^{-3}\ \text{radians/s})^2 = 1.101\text{x}10^3\ m/s^2$ = *112 [g's]* – apparently *still well within the weak-field approximation of Einstein's GW equations*.

Comparison of Results

From Eq. (A14) each of the components of force change, $\Delta f_{cfx,y} = 5.77\text{x}10^{32}$ N (multiplied by two since the centrifugal force reverses its direction each half period) and Δt = (1/2)(7.75hrx60minx60sec) = $1.395\text{x}10^4$ s for the half period. Thus using the jerk approach:

$$P = 1.76\text{x}10^{-52}\{(2r\Delta f_{cfx}/\Delta t)^2 + (2r\Delta f_{cfy}/\Delta t)^2\} = 1.76\text{x}10^{-52}(2\text{x}2.05\text{x}10^9\text{x}5.77\text{x}10^{32}/1.395\text{x}10^4)^2\text{x}2 = 10.1\text{x}10^{24}\ \text{W} \tag{A15}$$

versus the result of $9.296\text{x}10^{24}$ W using Landau and Lifshitz's more exact two-body-orbit formulation given by Eqs. (1.1) and (1.2) of Baker (1967) integrated using the mean anomaly not the true anomaly as independent variable. The most stunning closeness of the agreement is, of course, fortuitous since due to orbital eccentricity there is little symmetry among the $\Delta f_{cfx,y}$ components around the orbit and there are small errors inherent in the approximations of Eqs. (A3a) and (A3b) and, of course Eq. (A5) leading to Eq. (A10). Nevertheless, since the results for GW power are so close, orbital-mechanic formulation compared to the utilization of a jerk, *the correctness of the jerk formulation is well demonstrated!*